TURBULENT FLOW AND HEAT EXCHANGE OF ELECTROCONDUCTIVE FLUIDS IN A MAGNETIC FIELD

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Flows of electroconductive fluids in a magnetic field occur in such apparatus as magnetohydrodynamic pumps and generators, plasma accelerators, various units to regulate the consumption and batching of metals, and others. The mutual orientation of the magnetic and hydrodynamic fields can be distinct. If the stream of electroconductive fluid intersects the lines of force of the magnetic field, i.e., there is a flow velocity component normal to the magnetic lines of force, then the origination of induced electrical currents and their associated spatial electromagnetic forces in the stream is possible. However, this possibility is realized only if there is a flow velocity gradient along the magnetic field. For example, there are no induced currents in an electroconductive fluid stream moving translationally (column flow in a tube with nonconductive walls) no matter what the field orientation. If a transverse magnetic field is directed along the short side of a slit in the flow of an electroconductive fluid in a plane, infinite slit, then the directions of the velocity gradient and the magnetic field coincide resulting in the origin of the electromagnetic interaction known as the Hartman effect. When the magnetic field is directed along the long side of the slit, the velocity gradient is directed across the field and there is no electromagnetic interaction between the stream and the magnetic field. The reasoning presented above refers mainly to laminar flows in tubes with nonconductive walls. If the walls are produced from electroconductive materials, then the induced currents can be closed outside the fluid stream and the classification presented for the magnetohydrodynamic flows in a transverse fields should be broadened substantially by the examination of different operating modes of such channels. It is hence customary to introduce the so-called channel load coefficient K, the ratio between the "external" and induced electrical fields, as a characteristic. The wall conductivity does not affect the velocity profile in the case of a plane channel, however, such quantities as the Joulean heat evolution and total hydraulic drag of the channel depend substantially on K. It is natural that there is no magnetohydrodynamic interaction in the case of a longitudinal field when the velocity vector of fluid motion is parallel to the magnetic induction vector throughout the flow domain. If the flow in a tube is turbulent, then its interaction with a magnetic field holds for any mutual orientation between the magnetic and hydrodynamic fields since there are always velocity pulsations normal to the field lines of force in a turbulent stream. The interaction mentioned will result in dissipation of the pulsation energy, i.e., the turbulence should be suppressed by the magnetic field.

The effect of a magnetic field on turbulence naturally results in a change in turbulent transfer of momentum, heat, and mass. Hence, the influence of the magnetic field on the hydraulic drag and turbulent heat and mass exchange should be expected even in the case of no interaction between the magnetic field and the average flow.

Hartman [1] performed the first theoretical investigation of magnetohydrodynamic flows, and also, together with Lasarus [2], conducted the first experimental research on the influence of a transverse magnetic field on turbulent flows. Hartman and Lasarus drew attention to the fact that the influence of the transverse magnetic field is not always manifested in an increase in the hydraulic drag of the channel. In some flow modes a diminution in the hydraulic drag is observed as the magnitude of field induction increases. Hartman and Lasarus explained this effect by the magnetic field quenching the turbulent velocity pulsations. In later experiments in which the drag coefficient was measured for a Hartman flow, the influence of a

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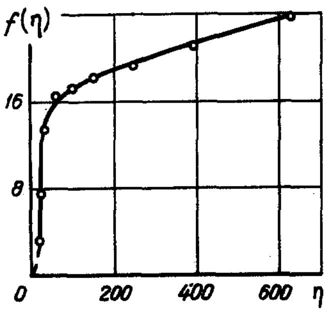

Fig. 1. The function $f(\eta)$.

transverse magnetic field on turbulence is also traced [3, 5]. However, its direct observation is made difficult because of the overwhelming influence of the Hartman effect.

In our opinion, flows in circular and plane tubes with nonconducting, smooth walls are most suitable objects for studying the mechanism of the influence of the magnetic field on turbulent flows of an electroconductive fluid. Hence, theoretical and experimental researches dealing with magnetohydrodynamic flows in tubes with electroconductive or rough walls will not be examined later. A survey of such research is contained in the monograph [6], wherein there is also a sufficiently detailed analysis of existing semiempirical theories. Hence, we limit ourselves to the analysis of some later achievements in the domain of investigation of turbulent magnetohydrodynamic flows and we present, wherever possible, the computational equations.

I. Semiempirical Theories of Turbulent Flows of an Electroconductive Fluid in a Magnetic Field

1. Turbulent Hartman Flow. The influence of a magnetic field on the hydrodynamic characteristics is composed of two effects in the case of turbulent Hartman flow: the suppression of turbulence and the increase in hydraulic drag because of reconstruction of the velocity profile due to the effect of the electromagnetic forces. The appropriate dimensionless equation of motion is [6]

$$\frac{d^2W_x}{dY^2} - \mathrm{Re}\frac{d(W_x'W_y')}{dY} + \mathrm{Ha}^2(1 - W_x) + \frac{\xi\,\mathrm{Re}}{2} = 0. \tag{1}$$

Here W_x, W_y are the ratios between the averaged velocity component and the mean velocity over the section, Re, Ha, ξ are the Reynolds and Hartman numbers and the friction coefficient, and the primes denote dimensionless velocity pulsations.

As is seen from this equation, the magnetic field can affect the turbulent momentum transfer (second term) and specifies the appearance of a spatial electromagnetic force (third term) in the stream. The total effect of these two operations on the stream is observed in measuring the friction. It hence turns out that for a sufficiently high value of Ha the third term in this equation is several orders of magnitude greater than the first and second. Hence, in practice any logical assumptions on the influence of the field on turbulent transport will result in fair agreement between theoretical and measurement results for the friction coefficient in this case [6]. On the other hand, for low values of Ha when the second and third terms are commensurate, the effect of the magnetic field on turbulence is slight, and the approximations of the turbulent tangential stress known in ordinary hydrodynamics will apparently be suitable to close the Reynolds equations. This circumstance is the reason why turbulent Hartman flow turned out to be the most appropriate object for the development of a semiempirical theory [6]. The semiempirical analysis of turbulent Hartman flow was carried out by many authors [6-10, etc.]. Let us briefly consider one semiempirical theory proposed by Branover [6]. The other researches have been analyzed in [6]. The author used the Prandtl expression for the turbulent friction

$$\tau_{\mathrm{T}} = -\overline{w_x'w_y'} = l^2\left|\frac{dw_x}{dy}\right|\frac{dw_x}{dy}.$$

The length of the mixing path was taken as

$$l = l_{00}\left[1 - \exp\left(-\alpha\,\mathrm{Re}\sqrt{\frac{\xi}{2}}\,Y\right)\right]\exp\left(-\beta S^2 \Big/ \frac{dW_x}{dY}\right), \tag{2}$$

where l_{00} is the Prandtl–Nikuradse mixing path length. The van Driest correction [30] is enclosed in the square brackets, and the second exponential factor takes account of the influence of the magnetic field on the scale of turbulence, which assures a smooth passage from ordinary turbulent flow with the Stuart number $S \to 0$ to the laminar mode (for $S \to \infty$ $l \to 0$). Moreover, the factor dW_x/dY takes account of nonuniformity in suppression of turbulence over the channel cross section.

Results of computing the hydraulic drag coefficient and the velocity profiles were compared with test data [2-5] by the author. The satisfactory agreement indicates the suitability of the assumptions made for

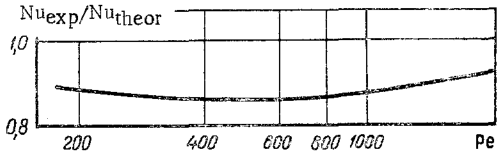

Fig. 2. Comparison between the results of experiment [37] and theory [38].

computation of turbulent Hartman flow. It is true that the comparison between the computed and experimental velocity profiles was made only for the stream core since the velocity measurements closest to the wall have as yet been made at a range of 0.05 of the channel half-width from the wall [5]. It is here pertinent to make other remarks also about existing test data on velocity profiles. Great discrepancies exist between the results of different authors relative to taking account of the influence of the magnetic field on the Pitot tube readings [11-14]. For example, for the Stuart number S = 0.5 computed by means of the hood diameter, the magnitude of the correction coefficient has a 70% discrepancy according to data in [11, 14], which results in a 30% discrepancy in the velocity values. Moreover, test data are ordinarily published in the form of dependences of the absolute value of the velocity on the distance from the wall, although it is well known that such a representation of the test data on a velocity profile does not permit an objective estimate of the spread in the experimental results and a convincing comparison of computation with test.

In conclusion, let us present the interpolation dependence proposed in [6] for the computation of the drag coefficient in turbulent Hartman flow in plane tubes with nonconductive walls

$$\xi = \xi_{Ha} + \left(\xi_0 - \frac{6}{Re}\right)\left(\frac{\xi_{Ha} - 0.00935}{6/Re - 0.00935}\right)^{1+\left(1-\frac{Re_{bo}}{Re}\right)^2} \quad (3)$$

Here $\xi_{Ha} = 2/Re \cdot Ha^2 thHa/Ha - thHa$ is the result of the Hartman solution for laminar flow, ξ_0 is the drag coefficient for a flow without a field, Re_{bo} is the boundary Reynolds number corresponding to $\xi_0 = 0.008$ for a flow without a field. The values of ξ, Re, and Ha should be computed relative to the halfwidth of the tube and the magnetic field direction.

2. Turbulent Flow in a Longitudinal Magnetic Field. The effect of the field on the flow is associated with suppression of turbulence in a turbulent flow in a longitudinal magnetic field. Hence, for satisfactory agreement with experiment, the semiempirical theory should take account of the influence of the field on turbulence more fully than in the case of Hartman flow.

Levin and Kovner [16, 17] obtained the most encouraging results, when they proposed semiempirical closure of the second-moment equations written for the flow in a magnetic field. Underlying this was the semiempirical method of Rotta, proposing an interpolation formula for total pulsation-energy dissipation and a relationship for energy exchange between the pulsation components along different coordinate axes [18], which permits closing the original system of equations. Levin [15] supplemented the Rotta method by obtaining the possibility of computing the turbulent stream characteristics down to the wall itself. The results of analyzing turbulent flows without a magnetic field in [15] agreed well with existing test data. This method was used in [16] for a semiempirical analysis of the turbulent flow in a magnetic field. Just one additional hypothesis related to the influence of the magnetic fields would hence be involved. The following hypothetical connection between the induced and "external" electrical field pulsations was taken:

$$e_r = -\beta \varepsilon_{rlm} w'_e \bar{B}_m, \quad (4)$$

where $0 < \beta < 1$, e is the pulsation of the electrical field, w' is the velocity pulsation, $\bar{B}$ is the average magnetic field induction (equal to the induction of the applied field for $Re_m \ll 1$), ε_{rlm} is an antisymmetric unit tensor of the third rank.

Another means for taking account of the influence of the magnetic field was selected in [17]: Joulean dissipation in the form

$$\frac{\alpha\sigma B}{\rho} \overline{w'_i w'_i}, \quad (5)$$

was added to the terms of the equation corresponding to viscous dissipation of the pulsation energy, where α is an empirical constant, σ and ρ are the fluid conductivity and density. The equations closed in this manner were solved under assumptions of the smallness of the turbulent and viscous diffusion of the pulsation energy. It must be noted that this latter assumption is not needed to close the equations and is taken only to simplify the computations.

A comparison of results of a computation by a method proposed in [17] with measured velocity profiles [19] is made in [20]. Satisfactory agreement is observed between the computation and the experiment, especially if it is taken into account that the method of computation was proposed prior to publication of the test results. It is true that attention was drawn to some discrepancy in the nature of the experimental and theoretical curves: near the tube walls the theoretical curves are above, and near the center below, the experimental curves. The reason for this discrepancy should apparently be sought in not taking account of diffusion of the pulsation energy. This latter proposition is verified by [21], where diffusion is taken into account in solving the pulsation energy equation in the absence of a magnetic field, essentially permitting the results of computing the local turbulent characteristics to approach the experimental results. Taking account of diffusion is apparently one of the possible means of perfecting this semiempirical theory, whose possibilities have not been exhausted by far.

Another means of approximating theory to experiment is to take account of the influence of the field on the scale of turbulence. The most general physical considerations [6] suggest that the spectrum of turbulence should shift towards higher wave numbers under the effect of a magnetic field. This can be taken into account by postulating some diminution in the scale under the effect of the field.

Some inadequacy in the experimental results is also observed in the case of a longitudinal field. Although the influence of the longitudinal magnetic field on the hydraulic drag coefficient has been studied sufficiently well [19, 22-25], the velocity profiles have been measured only in [19]. The measurements included 98% of the tube radius and the results are represented in universal coordinates, which is convenient for a comparison with theory.

The interpolation equation proposed in [19]:

$$\frac{\xi_L - \xi}{\xi_T - \xi} = \frac{0.173X^{2.24}}{1 + 0.173X^{2.24}}, \tag{6}$$

where $\xi_L = 64/\mathrm{Re}$, $\xi_T = 0.316/\mathrm{Re}^{0.25}$, $X = \mathrm{Ha}/0.1\,(\mathrm{Re} - 2300)^{0.77}$; the numbers Re, Ha, and ξ are computed relative to the tube diameter, can be recommended for computing the hydraulic drag coefficient for turbulent flow in a circular tube in a longitudinal magnetic field. For values $X \le 2.4$, Eq. (6) describes the test data in [19] to 3% accuracy.

For turbulent flow in a circular tube in a longitudinal magnetic field the velocity field can be described by the equation [20]

$$\frac{w}{u^+} = \left(\frac{w}{u^+}\right)_{\mathrm{Ha}=0} + f(\eta)\left(\frac{\mathrm{Ha}}{\mathrm{Re}^+} - 0.13\right), \tag{7}$$

where u^+ is the dynamic velocity, $\mathrm{Re}^+ = u^+d/\nu$; $\mathrm{Ha} = Bd\sqrt{\sigma/\rho\nu}$; $\eta = u^+y/\nu$; ρ, σ, ν are the density, electrical conductivity, and kinematic coefficient of viscosity of the fluid, d is the tube diameter, and y is the distance from the tube wall. The function $f(\eta)$ is presented in Fig. 1. The equation is valid for $\mathrm{Ha}/\mathrm{Re}^+ \ge 0.13$.

3. Turbulent Flow in Plane Tubes with the Long Side of the Cross Section along the Magnetic Field. As has already been remarked in the introduction, interaction between the averaged flow and the magnetic field should not occur for the transverse magnetic field oriented along a plane infinite slot. A good approximation to this geometry in experiment can be achieved by using plane tubes with a large ratio between the sides of the cross section. The drag coefficients in such tubes, oriented with the long side of the section along the magnetic field, were measured in the Institute of Physics of the Academy of Sciences of the Latvian SSR [26, 27]. New interesting effects were hence detected. The reduction in hydraulic drag turned out to be quite strong (5-7 times stronger than for a longitudinal magnetic field). The authors explained this difference by the fact that the transverse magnetic field acts directly on the longitudinal component of the velocity pulsation, which is larger in amplitude and scale than the transverse components with which the longitudinal field interacts. In our opinion, still another consideration associated with the transmission mechanism of energy pulsations of the averaged flow should be added. It can be shown [29] that the energy of the averaged motion is transmitted directly only by the longitudinal component of the pulsations. As regards the transverse velocity pulsations, they receive energy only from the longitudinal pulsations. Hence, it can be assumed that the effect of the transverse field directly on the longitudinal velocity pulsations will result in stronger laminarization of the flow than in the case of a longitudinal field. The effect of flow laminarization was detected directly in measuring the velocity profile at the center of the tube [28]. The

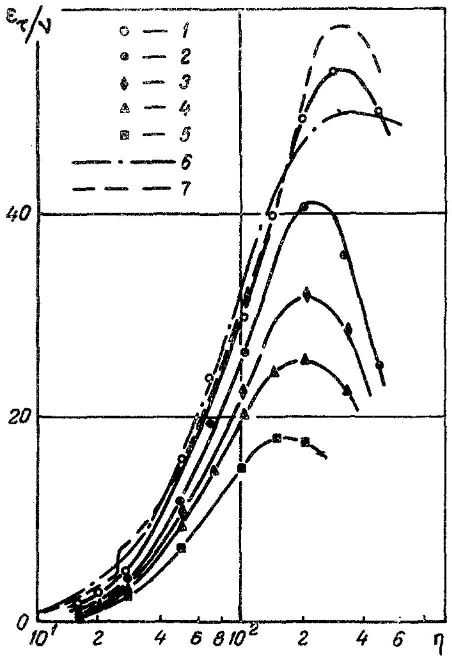

Fig. 3. Coefficient of turbulent momentum transfer for Re = $2.39 \cdot 10^4$ [40]: 1) Ha = 0; 2) 270; 3) 390; 4) 502; 5) 614; 6) from Reichardt [48]; 7) from Deisler [49].

measurements showed the rise in velocity with the growth in the Ha number. Branover and Vasil'ev [31] developed a semi-empirical theory of such a flow. It is interesting to note that good agreement with experiment was successfully achieved by using a hypothesis analogous to (2) about the influence of the field on the mixing length, which was written in this case as

$$l = \frac{l_0}{1 + \varkappa (S l_0)^{3/2}}, \quad (8)$$

where $S = Ha^2/Re$ is the Stuart number, $l_0 = l_{00}[1 - \exp(-\alpha \cdot Re\sqrt{\xi/2}\,Y)]$ is the mixing path in the absence of a field according to van Driest [30], and l_{00} is the Prandtl–Nikuradze scale.

Good agreement between the hydraulic drag computations and test results was successfully achieved by selecting the empirical constants α and $\varkappa$. It is true that the authors of [31] do not present a comparison between the results of computing the velocity profile and the results of the proper experiments in [28], which would permit a more objective estimate of the proposed method of computation. An interpolation equation to compute the hydraulic drag coefficient

$$\xi/\xi_0 = 1 - 1300\,(Ha/Re)^{1.6}, \quad (9)$$

where ξ_0 is the hydraulic drag coefficient in the absence of a field, has been proposed [32] for the domain far from total flow laminarization. This equation is valid for the values Re/Ha > 130.

II. Heat Exchange in the Turbulent Flow of an Electroconductive Fluid in a Magnetic Field

The heat exchange in a turbulent flow in a magnetic field has been studied relatively little at the present time. There are only five experimental papers on this question [34, 35, 37, 38, 42]. The problem of investigating the influence of a magnetic field on heat exchange in a turbulent flow was apparently first posed therein. The heat emission in the flow of an air plasma in a circular tube in the field of a solenoid was measured. A sufficiently unique influence of the magnetic field on the heat exchange was hence detected which is dependent not only on the field intensity but also its direction. This effect is appraently due to twisting of the flow in an electric-arc heater, in which connection the flow in the tube can be spiral and the observed influence of the field on the heat emission is not related to suppression of turbulence but to the effect of the field on the averaged flow. Moreover, the heat emission was measured in [35] on the thermal and hydrodynamic stabilization sections, which makes analysis of the test data difficult in the absence of accurate data on the conditions at the entrance to the working section.

1. Heat Exchange in Turbulent Flow in a Transverse Magnetic Field. The heat emission in the stabilized turbulent flow of gallium in a rectangular channel of 40 × 6 mm^2 cross section placed in a transverse field directed along the short side of the section was measured in [38]. The measurements were made in the range of Re numbers $9 \cdot 10^3$–$1.25 \cdot 10^5$ for three values of the Ha number: 60, 90, and 120. The maximum reduction in the heat elimination by the magnetic field was ~30% and was observed at values of Re = (5-8) $\cdot 10^4$. The author generalized the experimental results by the following dependence:

$$Nu_{exp} = 9 + \frac{0.006Pe}{1 + 14.8S}. \quad (10)$$

This equation is applicable in the Ha number range between 0 and 120 and the Re number range between 180 and 2500.

The results of investigating the heat emission in a turbulent mercury flow in a circular tube in a transverse magnetic field are presented in [42]. The measurements were performed in a 4.6 mm diameter stainless steel tube for values 0 and 115 of the Ha number in the 9000-137,000 range of Re numbers. The maximum reduction in heat emission by the magnetic field, just as in [37], was around 30% and was observed for the values $Re = (5-8) \cdot 10^4$. The authors did not present the interpolation equation.

Finally, the heat emission to a stream of electrolyte (15% aqueous solution of KOH) in a rectangular tube of 70×23 mm² cross section was measured in [34]. The measurements included the $2 \cdot 10^3 - 3 \cdot 10^4$ Re number range for the number Ha varying between 0 and 10. The maximum influence of the magnetic field on the heat emission did not exceed 10% and was observed at the value Re = 4100. The author proposed the interpolation equation

$$Nu = Nu_0 (1 - 3S), \tag{11}$$

for developed turbulent flow in a transverse magnetic field (for $Re > Re_{cr}$).

For $Re > 1.5\, Re_{cr}$ this equation describes well both the author's results and the results in [42].

The following deductions can be made in concluding the analysis of the fundamental experimental results relative to heat exchange in a turbulent flow in a transverse magnetic field. The nature of the influence of the field on heat exchange depends on the Re value, and a range of Re numbers wherein this influence is a maximum has been detected. This phenomenon has a simple physical meaning. Indeed, the diminution in heat exchange under the effect of a magnetic field is associated with suppression of turbulence. It is also known that the contribution of turbulent heat transfer to the total heat flux grows from zero in laminar flow to ~100% as $Re \to \infty$. Moreover, the degree of turbulence suppression diminishes as Re grows because of the diminution in the scale of large-scale vortices responsible for molar transfer. The total effect of these three factors absolutely results in the dependence $Nu/Nu_0 = f(Re)$ having a minimum for any values of the numbers Pr and Ha. These considerations also refer completely to heat exchange in a turbulent flow in a longitudinal magnetic field. The value of Re at which the maximum influence of the magnetic field on heat emission is observed should depend on the Ha number and the flow geometry, i.e., the factors affecting the degree of stream laminarization. The maximum influence of the magnetic field on heat exchange will apparently be observed for similar Re values for flows whose degree of laminarization are identical. This deduction is verified by a representation of the test data in the form of the dependence

$$Nu/Nu_0 = f(Re/Re_{cr}), \tag{12}$$

where Re_{cr} corresponds to the transition from turbulent to laminar magnetohydrodynamic flow. Both for the electrolyte and for mercury, the minimum of the dependence (12) is observed at similar values of the ratio Re/Re_{cr}.

As the experimental investigations examined above have shown, the observed reduction in heat emission in a transverse magnetic field did not exceed 30% when liquid metals and electrolytes were used as working fluids, although the field induction reached sufficiently high values (0.98 Wb/m² in [42] and 2.1 Wb/m² in [34]). Unfortunately, liquid metals and electrolytes are the only fluids, at present, in which sufficiently exact measurements of the effect of a magnetic field on the stabilized heat exchange can be performed, although a working fluid with both high electrical conductivity and a high value of the Pr number is necessary from the viewpoint of the maximum appearance of the effect of field reduction of the heat emission. Liquid metals and electrolytes possess only some of these advantages. An attempt [55] to investigate the heat exchange in a magnetic field in the transition region ($Re \le 4200$) by using argon with a potassium supplement as working fluid is known. However, in this case the observed effect was within the limits of measurement error. At present it is impossible, unfortunately, to indicate an accurate criterion for the suitability of a fluid as a heat carrier in an investigation of the heat exchange in a transverse magnetic field. It can only be asserted that for a given Pr value a fluid having a maximum ratio of the conductivity to the dynamic coefficient of viscosity is more suitable. This question can be resolved somewhat more definitely in the case of a longitudinal field (§II, 2).

The stronger influence of a transverse magnetic field on heat emission should be expected when the field is oriented along the long side of the section of a flat tube. This is related, firstly, to the stronger laminarization of the flow by such a field (§I) and, secondly, to the absence of the Hartman effect which partially cancels the reduction in heat emission because of the quenching of turbulence [50]. However, there are as yet no such investigations.

A semiempirical analysis of the heat exchange in turbulent Hartman flow was first performed by Krasil'nikov [36, 38]. The heat emission was computed by using the known Lyon integral relation [45]

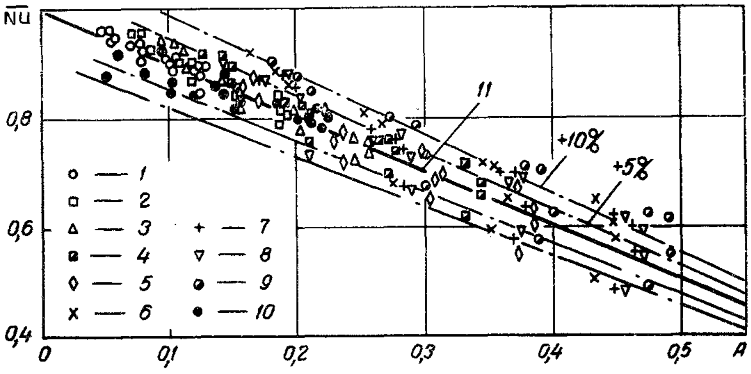

Fig. 4. Generalization of the results of a computation [40]: 1) Pr = 0.005; 2) 0.01; 3) 0.02; 4) 0.05; 5) 0.10; 6) 0.30; 7) 0.70; 8) 1.0; 9) 3; 10) test data [37]; the line 11 is Eq. (18); and A is the second member in the right side of (18).

under the assumption $Re_m \ll 1$ and $Pr_{turb} = 1$. The expression for the velocity profile and data on the coefficient of turbulent momentum transfer were taken from [46], where a solution has been obtained for the problem of turbulent flow in a transverse magnetic field on the basis of the generalized Loitsyanskii locality hypothesis. The author approximated the results of a numerical computation of the heat emission by the dependence [36, 38]

$$Nu_{theor} = 10 + 0.025\left[Pe\left(\frac{1}{1+14.8S}\right)\right]^{0.8}, \qquad (13)$$

where Nu, Pe, S are the Nusselt, Peclet, and Stuart numbers.

Another attempt at a theoretical analysis of the heat emission in a turbulent flow in a transverse magnetic field is due to Novikov [43, 44], who used the hypothesis of conservation of a power-law dependence for the velocity profile in a magnetic field, which had been approved in ordinary hydrodynamics:

$$w_x = w_0\left(\frac{y}{\delta}\right)^{\frac{m}{2-m}}, \qquad (14)$$

where w_0 is the velocity on the outer boundary of the boundary layer, y is the distance from the wall, and δ is the boundary layer thickness. A dependence of the boundary layer thickness δ and of Nu_x on the numbers Re and Ha is obtained by using (14) for the turbulent flow around a flat plate in [43].

The Nu number for stabilized flow in a tube [44] has been obtained by passing to the limit as $x \to l_{its}$ (where l_{its} is the length of the initial thermal section) in the form

$$Nu = Nu_0\left(1 - n'\frac{Ha^2}{Re^{1-m}}\right), \qquad (15)$$

where m = 0.25, n' is an empirical constant, and Nu_0 is the Nusselt number without a magnetic field. Let us note that the author obtained the dependence $Nu_0 \sim Re^{0.75}$ in the absence of a magnetic field. The author recommended (15) for computation of the heat exchange under the condition $Ha \gg 1$; the range of Pr numbers within which (15) is applicable is not stipulated in [44].

In structure, (15) is similar to the empirical dependence (11) proposed in [34] to compute the heat emission in a transverse field in the case of a developed turbulent stream. The constants in (15) should hence be taken equal to n' = 3, m = 0.

In conclusion, let us present (Fig. 2) a comparison between the semiempirical Krasil'nikov formula (13) and its test data [37], which the author approximated by (10). It is seen from the figure that for Ha = 90 the experimental values of the number Nu turned out to be 10-15% below the theoretical values. In our opinion, two reasons, besides the imperfection of the theory, are possible for the discrepancy between theory and experiment. As the author himself remarks, first the influence of contact thermal resistance on heat emission is possible. Second, the computation is performed for nonconducting tube walls, and the heat emission is measured on a copper working section, which could result in a reduction in heat emission because of the large influence of Joulean heat liberation in the case of conducting walls.

2. Heat Exchange in the Turbulent Flow of an Electrically Conducting Fluid in Tubes in a Longitudinal Magnetic Field. Besides [35], which is briefly analyzed at the beginning of this section, there is still another paper [37] devoted to an experimental investigation of heat exchange for turbulent flow in a tube in a longitudinal magnetic field. The authors of [37] use liquid gallium as the working fluid. The heat emission was measured in circular copper tubes with 16.9 and 19.6 mm diameters. The working sections of the tubes were placed in the magnetic field of a solenoid with up to 0.75 Wb/m^2 as the maximum field induction. The measurements included the 160-3100 Pe, 0-550 Ha, and $8 \cdot 10^3$-$1.5 \cdot 10^5$ Re number ranges. The results of the measurements are generalized well by the empirical dependence

$$\mathrm{Nu} = 6.5 + \frac{0.005\mathrm{Pe}}{1+1890\,(\mathrm{Ha}/\mathrm{Re})^{1.7}}. \tag{16}$$

Exactly as in the case of the transverse magnetic field, a domain of Re numbers is observed in which the effect of the magnetic field on the heat emission is a maximum (§II, 1). A maximum reduction in the heat emission (~25%) was observed at Re ≃ 40,000 for the value Ha = 550.

Krasil'nikov [38, 39] also computed the heat emission in a turbulent flow in a longitudinal field. As in the case of the transverse field, the Lyon integral relation and the assumption that $\mathrm{Pr}_{turb} = 1$ were used. In conformity with [47] the velocity profile was taken as

$$\frac{w}{u^+} = \left(\frac{w}{u^+}\right)_{\mathrm{Ha}=0} + A\eta\left(\frac{\mathrm{Ha}}{\mathrm{Re}^+}\right)^2, \tag{17}$$

where u^+ is the dynamic velocity, η is a universal constant, $\mathrm{Re}^+ = u^+d/\nu$, and A is an empirical constant. The results of a numerical computation, according to the author, yield an exaggerated influence of the magnetic field as compared with test. This is not surprising since a comparison between the velocity profile (17) used in the computations and (7) obtained in [20] on the basis of experimental results shows that the influence of the field on the velocity profile is highly exaggerated in (17), indeed, the second member in the right side is linear in $\mathrm{Ha}/\mathrm{Re}^+$.

The Lyon integral relation and the assumption that $\mathrm{Pr}_{turb} = 1$ were also used in [40] to compute the heat exchange in a turbulent flow in a longitudinal magnetic field. However, the experimental results obtained earlier about velocity profiles [19], whose differentiation defined the coefficient of turbulent momentum transfer, were used in the computations. Presented in Fig. 3 as an illustration are the results of computing ε_T/ν for the value $\mathrm{Re} = 2.39 \cdot 10^4$ for five values of the number Ha. It is seen from the figure that in the absence of a magnetic field the results of the computation are in good agreement with the known dependences of Reichardt and Deisler [48, 49], and application of a magnetic field essentially diminishes the turbulent transfer. The heat emission was computed for four values of the Re number (from $2.39 \cdot 10^4$ to $4.25 \cdot 10^4$), five values of the Ha number (from 0 to 614), and ten values of the Pr number (from 0.005 to 3). The results of a computation for Pr = 0.02 (liquid gallium) agreed well with the Krasil'nikov test data (Eq. (16)). The results of the computation were extrapolated from the $2 \cdot 10^3$ to the $1.5 \cdot 10^5$ band of Re numbers by using (16) as a reference dependence. The fact was hence used that the value of Re at which the maximum influence of the field on heat emission is observed in the case of constant fluid properties is independent of the Pr number, but is determined only by the Ha number and the flow geometry as in the case of the transverse field. The interpolation equation

$$\overline{\mathrm{Nu}} = \mathrm{Nu}/\mathrm{Nu}_0 = 1 - \mathrm{Ha}^{0.67}(\mathrm{Re} - 2300)^{0.75} C_1 (\lg \mathrm{Pr} + C_2) \exp(-5.2 \cdot 10^{-4} \mathrm{Re}/\mathrm{Ha}^{0.48}), \tag{18}$$

where C_1 and C_2 are constants dependent on the Pr number range:

$$C_1 = 2.77 \cdot 10^{-6};\ C_2 = 2.90 \text{ for } \mathrm{Pr} \leqslant 0.1,$$
$$C_1 = 5.44 \cdot 10^{-7};\ C_2 = 11.2 \text{ for } 0.1 < \mathrm{Pr} \leqslant 3.$$

was obtained on the basis of test data in [37], the results of a computation, and their extrapolation. For Pr = 0.02, Eq. (18) agrees with (16) to 2% accuracy.

A comparison between (18) and test data in [37] and the results of a computation is presented in Fig. 4. Equation (6) can be recommended for the computation of the heat emission in a stabilized turbulent flow in a circular tube in a longitudinal magnetic field with a constant heat flux density along the tube length in the 0-3 Pr number range, $2.3 \cdot 10^3$-$1.5 \cdot 10^5$ Re number range, and 0-614 Ha number range. In order to compute the value of Nu it is first necessary to determine Nu_0. In our opinion, the value of Nu_0 in the $\mathrm{Pr} \leq 0.1$ band should be calculated from the formula

TABLE 1

Working fluid	15% KOH	Mercury	$N_2 + 1\%$ K	Ar + 1% Cs
$B_0 = 1$ Wb/m², $L_0 = 0.02$ m				
t_f, °C	90	100	2500	2000
Ha_0	9,04	557	12,5	29,8
Pr_0	2,63	0,019	0,702	0,667
Re^*_{max}	6450	32500	7150	9660
Nu/Nu_0	0,995	0,768	0,994	0,983

* Re_{max} is the value of Re for which the maximum influence of the field on heat emission should be expected according to (18).

$$Nu_0 = 5 + 0.025\,Pe^{0.8}, \tag{19}$$

which has the best foundation at the present time [51]. For values of $Pr \geq 0.5$ the equation proposed in [51] can be used to compute Nu_0. Let us note that an analogous heat-exchange computation can also be performed for turbulent flow in a transverse magnetic field after sufficiently detailed data have been obtained on the velocity profiles.

Equation (18) permits a more definite solution than in the case of the transverse field, of the question of the most suitable working fluid for an experimental investigation of the heat exchange in a magnetic field. By using the data presented in [33] we compared an electrolyte solution, mercury, and inert gases with an admixture of alkali metals from this viewpoint (Table 1). It is seen from Table 1 that the most appropriate working fluids in the case of a longitudinal field are the liquid metals. It is quite possible that this deduction is also valid for turbulent flow in a transverse field.

More significant effects can apparently be observed in an investigation of the mass exchange in a magnetic field since large values of the Schmidt number can be assured. However, the carrying over of results about mass exchange to heat exchange is not always possible since the use of a known analogy between heat and mass exchange in the presence of a magnetic field is limited. The constraints are related to the fact that Joulean heat liberation in a stream additionally influences heat exchange without exerting influence on the mass exchange. It is true that according to the computations of the authors of [33], Joulean dissipation in liquid-metal streams starts to influence heat exchange noticeably in a transverse field at values of $Ha \sim 10^2$. Therefore, the analogy mentioned should be valid in a sufficiently broad band of Ha. As regards the flows in a longitudinal field, no constraints are foreseen on the use of the analogy between heat and mass exchange since the Joulean dissipation in the pulsating currents is always less than the viscous dissipation, and as is known, this latter is negligible in the case of fluid drops for all flow velocities achievable under laboratory conditions.

The investigation of mass exchange in a turbulent flow in a magnetic field is therefore of great interest from the viewpoint of modelling the essential effects of the magnetic field on heat exchange. Unfortunately, there are no such investigations as yet. However, the recently published papers [53, 54] on an investigation of mass exchange in laminar magnetohydrodynamic flows permit the hope that interesting research on mass exchange in a turbulent flow in a magnetic field will appear in the near future.

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